Liquidity risk, liquidity demand of investors and asset pricing

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Among the field of asset pricing theory, the theoretical significance of market liquidity risk premium is a hot topic. This paper decomposes market liquidity risk into exogenous and endogenous liquidity risk, and introduces liquidity demand as a state variable, giving rise to the random holding horizon, and develops a liquidity risk-adjusted capital asset pricing model. Besides agreement with the previous theoretical literatures about the effect of exogenous liquidity risk on asset pricing, we find that different elasticity value of price impact can make a cross-sectional dispersion in required return for the level of liquidity and market liquidity risk. The state variable of liquidity demand affects market liquidity risk premium increasingly, and could induce the known time-varying phenomenon of liquidity risk premium.

Key words: Liquidity risk, liquidity demand, asset pricing.

INTRODUCTION

Liquidity is an asset’s ability to be sold without causing a significant movement in the price and with minimum loss of value. More liquid asset leads to easier transaction, more stable price and low transaction cost. The unpredictable change in liquidity could be called liquidity risk. For investor, real market is far from the perfect market with no transaction cost and everlasting equilibrium from classical asset pricing model. So, it is reasonable to take liquidity and liquidity risk into consideration. However, the relation between liquidity risk and theoretical meaning for asset pricing has been ignored; the threshold of this relation just comes from market micro-structure theory from Amihud and Mendelson (1986). Some Chinese researchers, such as Su and Mai (2004), have also contributed to this relation. Although market micro-structure theory confirms the significant influence to stock return, it makes little progress for liquidity risks. The reason behind is that the traditional market micro-structure theory focuses on individual stock instead of the systematicness of liquidity, but the relation between liquidity risk and pricing should separate systematic risk from specific risk, only systematic risk can influence asset price from classical financial theories. Chordia et al. (2000), Hasbrouck and Seppi (2001), and Huberman and Halka (2001) did some empirical studies on systematic risk on stock liquidity, which brought the hot point from individual liquidity to the systematicness of liquidity, including the significant relation between liquidity risk and asset pricing from empirical researches by different academics.

There are few papers on this mentioned relation; Acharya and Pedersen (2005) designed an exogenous liquidity risk-adjusted capital asset pricing model, but this paper ignored another important part: endogenous liquidity risk, and cannot explain time-varying phenomenon of liquidity risk premium. Based on these flaws, the substance of this study is to introduce endogenous liquidity risk to expand a new liquidity risk-adjusted capital asset pricing model from the ubiquity of direct connection between transaction cost and the volume of trading position in security market, especially order-driven market. In this new model, the investor’ demand for liquidity has been added. Whether the liquidity

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demand tension is directly related with the preference of investor asset liquidity would be reflected in required liquidity premium, which can explain time-varying phenomenon of liquidity risk premium.

LITERATURE REVIEW

Chordia et al. (2000) had asserted that if liquidity shock in market cannot be diversified, stock which is more sensitivity to the total market liquidity would be required more return. Pastor and Stambaugh (2003) gave the earliest positive respond. They set the liquid $\beta$ as the individual stock sensitivity ratio to the market portfolio liquidity perturbation. Based on the data in U.S stock market from 1966 to 1999, they discovered the average annual return of portfolio with highest liquid $\beta$ is 7.5% higher than that of the portfolio with lowest liquid $\beta$, after adjusting for the three factors of Fama-French and momentum factor. Eckbo and Norli (2002), Wang (2003), Sadka (2004), and Acharya and Pedersen (2005) also got the similar stock market cross-section result. For example, Acharya and Pedersen (2005) who did empirical research on the performance of U.S, NYSE and AMEX during 1963 and 1999, found that the $R^2$ in liquidity risk-adjusted CAPM is much higher than classical $R^2$. Based on adjusted CAPM, adding market $\beta$ and three $\beta$ on liquidity together, the total $\beta$ had a significant premium. Avramov et al. (2002) found that the portfolio with liquidity as one risk factor is more close to multi-factors unbiased variance frontier by Merton's ICAPM. So adding liquidity risk into consideration would diminish pricing error.

Acharya and Pedersen (2005) made up a liquidity risk-adjusted asset pricing model to explain liquidity risk premium in empirical study and analyze the mechanism of action for asset pricing. In a two-period iterative model, they set two assumptions. Firstly, investors had a negative index utility function, with the absolute risk-adverse coefficient equals to a constant. Secondly, the distribution of dividends and transaction costs are first-order autoregressive conditional normally distributed separately. This model showed that the undiversified part of liquidity risk, which performed as three covariance forms, is systematic liquidity risks, and influenced the required return of assets. The three covariance are following: 1) covariance between security liquidity and market portfolio, $\alpha(x,c^{\prime})$; 2) covariance between security return and market portfolio liquidity, $\alpha(x,r^{\prime})$; 3) covariance between security return and market portfolio return, $\alpha(x,r)$. This model confirmed the empirical meaning of liquidity of the works by Chordia et al. (2000) and explained the empirical conclusion by Pastor and Stambaugh (2003), who argued that liquidity sensitivity is significantly priced and added into return. These viewpoints made us get a relatively uniform idea on liquidity risk pricing mechanism.

However, Acharya and Pedersen (2005) ignored two crucial problems. One is endogenous liquidity risk in transaction. They assumed that the transaction cost is first-order autoregressive process and absolutely exogenous, which had no relation with the amount of position. It is obviously not in conformity with actual condition. Bigger amount position would make it more difficult to make a deal and incur a higher average transaction cost. So, the optimal liquidation strategy existed. We can conclude that rational investors would prefer to add the appropriate amount of position into the investment decision and security pricing, and this approach has the implication of the future liquidation cost.

The emphasis on endogenous liquidity risk is compatible in China market, because China market is an order-driven market, which has no market dealers to supply liquidity service. In order-driven markets, investors need to find counterparty by themselves, and it is very common to make a deal in several different price level, which enhances the difficulty and uncertainty and makes investors focus more on the ‘optimal amount’ of the position. It is much more important for institutional investors, who could exert more influence on the price. With institutional investors get more and more share in the total market participants in China market, it is found that large-cap stocks are preferred by funds and other institutional investors. From liquidity perspective, the reason is large-cap stocks have smaller shock cost and smaller endogenous liquidity risk.

Another problem ignored by Acharya and Pedersen (2005) is endogenous holding period. They assumed that investors purchase stock in first period and sell all in second period. It would overestimate the liquidity transaction cost if the holding period is determined exogenously. The reason is that investors could avoid less liquidity period to choose better opportunity, or diminish transaction frequency, or be more patient to sell portfolio gradually instead of one-off to diminish the negative influence from liquidity. This assumption does not hold the opinion that investors are immunized against market liquidity perfectly, because investors have limited borrowing capacity and pressure on liquidity cash demand, which makes liquidation time become less controllable and random (Huang 2002; Lynch and Tan, 2004). So the motivation to diversify risks makes investors choose their market liquidity preference on their liquidity demand state.

economic recession index. Fujimoto and Watanabe (2005) using data of individual stocks between 1965 and 2004 in U.S market to verify time-varying phenomenon of liquidity risk premium. They discovered the distribution of the β of individual stock and liquidity risk premium is distributed in a high-low binary regime-switching. High β and high liquidity risk premium is the state in which investors prefer more liquid market and the investors’ liquidity demand is tense.

As a substitution for Acharya and Pedersen’s (2005) model, we would set a released binary two-period iteration model. As an Introduction, liquidity demand state variables to make the holding period become random, and adding endogenous liquidity risk, we follow their path to use transaction cost to reflect liquidity. Adding endogenous liquidity risk would underline liquidity factor for asset pricing. Although the endogenous problem of holding period cannot be solved by randomization, the overestimation problems can be solved successfully. At the same time, our model can explain the time-varying phenomenon of liquidity risk premium, which cannot be explained by the model set by Acharya and Pedersen (2005). The idea of introduction liquidity demand state variance is in line with the idea of Holmstrom and Tirole (2000), who set a corporate liquidity demand asset pricing model based on corporate finance. In this model, the assumed that firms need to meet the future liquidity cash uncertain demand in multi-period production, Therefore, firms had the demand for liquid financial asset and liquidity premium, which is related with the covariance of asset future return and marginal liquidity value. It is worth mentioning that our model is from optimization on investor’s consumption-investment relation, to research market individual liquidity, total liquidity and market liquidity risk premium based on uncertain liquidity demand for investors. While the works of Holmstrom and Tirole (2000) start from investors diversifying liquidity risk analyzing liquidity premium, but ignoring security market endogenous liquidity risk.

METHODOLOGY

Introduction of the modeling

Market

Suppose that discrete time t equals to -1, 0, 1……., only one consumable product exists in each period. The price of consumable product is standardized to 1 as the price system base. There are J kinds of securities in capital market; the number j security has the supply $S^j$. In each period, each unit security has the dividend $D^j_t$, which is in the markoff process, and the price of this security is $P^j_t$. Meanwhile, it is the order that drives the market. The liquidity of security reflects the transaction cost, better liquidity lead to a smaller cost, and vice versa. So, it can be called the liquidity transaction cost. Accurately, real security transaction cost should include fixed cost, price shock cost, and time-used cost and so on. Here, we just use the loss $q^j$ as shock cost of striking price as the general proxy, which is in the markoff process, reflects the transaction cost of per unit security. Because the shock for the striking price would grow with the amount of transaction amount, it could be assumed that $q^j$ is the increasing function, which has the second partial derivative. In our model, the transaction cost of each unit is the random variable in the markoff process, which expresses idea of the exogenous liquidity risk. While time-varying unit transaction cost is the increasing function of transaction amount, which reflects the endogenous liquidity risk.

Investor

In a simple two-period iterative model, for simplicity, it is assumed that $N_t$ homogeneous investors were born in t period, the time preference is constant number $\rho$, and the utility function is second differentiable concave function $U(C_t)$. Meanwhile, $E(U'(C_{t+1}))$ exists, and $C_t$ is the consumption of t period. The proxy of initial endowment is e, they would enter into capital market to purchase securities, the vendor of purchasing are $x_t = (x^1_t, x^2_t, ..., x^J_t)'$. At t+1 period, securities would be sold for the new-born investors.

It is not necessary that investor pay for $q^j_{t+1}(x_t)$ transaction cost, which depends on current liquidity demand state. Suppose that two outcomes of liquidity demand are possible. One is stress state, investor has to dump all securities to meet the liquidity demand, so the transaction cost is $q^j_{t+1}(x_t)$, and the probability is $\lambda_{t+1}$. Many reasons may lead to stress state, including a family faces a sudden wealth loss, an institution need to chase an investment opportunity or need to change the portfolio, or hold more money to meet unexpected redeem. Another outcome is relaxed state with the opportunity equaling to $(1-\lambda_{t+1})$, in which investor can be immunized against market liquidity effect, do not need take the transaction cost of $q^j_{t+1}(x_t)$. For example, investor has no external demand for cash or other opportunity cost in current period, so they can split securities until the cost is too small to be ignored, or they can hold to next period. Anyway, transaction cost can be regarded as zero, which could be seen that this investor sells all his securities to himself, a new investor in the next period. A new indicative function would express two possible liquidity demand state variables:

$$I_{t+1} = \begin{cases} 1, \text{liquidity demand is stress state} \\ 0, \text{liquidity demand is relaxed state} \end{cases}$$

Randomizing the transaction cost of investor at the end of the investment term can release and randomize holding period. The probability is related with the length of measuring time, the state of market and macroeconomic situation. A longer time, more uncertain market and macroeconomic situation would enhance $\lambda_{t+1}$. By the way, all variables would be defined in the probability space $(\Omega, F, P)$. 
Investor's optimization

The liquidity risk includes two parts: one is the uncertainty of liquidity demand, the other is liquidity risk of capital market. Within the risk tied condition, proxy investor optimizes his consumption-investment proposition to maximize his utility. In first period, investor decides his consumption-investment proposition based on initial endowment, and sells securities based on his personal liquidity demand in second period. The target is to maximize utility of the two periods:

\[ V(e_t, q_{t+1}, D_t) = \max_{x_t} \{ U(C_t) + E_t \rho U(C_{t+1}) \} \]

s.t. \[ C_t = e_t - P_t x_t \]

\[ C_{t+1} = (P_t + D_{t+1} - I_{t+1} q_{t+1}) x_t \]

So, the price of security follows the Euler equation including transaction cost:

\[ P_t = E_t \left\{ \rho U'(C_t) \left( P_t + D_{t+1} - I_{t+1} q_{t+1} - I_{t+1} \frac{\partial q_{t+1}}{\partial x_t} x_t \right) \right\} \]

So, the equilibrium price and investor's position would be set by (3) and (4). It is necessary to set the detailed form if the form of expected return is needed.

Liquidity risk-adjusted CAPM

We need to set some assumptions for price shock elasticity and utility function to linearize the asset pricing model in the last part for a liquidity risk-adjusted CAPM. Define \[ \alpha^j \equiv \left( \frac{\partial q_{t+1}^j}{\partial x_t^j} / \left( \frac{\partial q_t^j}{\partial x_t^j} \right) \right) \] as the shock cost elasticity of the transaction position, and obviously \[ \alpha^j > 0 \]. Chordia et al. (2001) and Hasbrouck and Seppi (2001) believed that the change in liquidity reflects the internal change in market activity instead of the volume of transaction position. For explication, we regard shock elasticity \[ \alpha^j \] as a variable which is not a time-varying but related with transaction position in the idea mentioned above. For example, in the market shock cost model of Torre (1997), the amount of liquidity premium would increase in the amount of square root of transaction position. If the market shock cost is the linear function of the square root of the transaction cost, \[ \alpha^j \] can be calculated to identically equal to 1/2. Investors' utility function can use common quadratic function:

\[ U_t(C_t) = - \frac{1}{2} (C_t - C_t^0)^2 \]

Or equivalently:

\[ E_t \left( r_{t+1} \right) = r' + (1 + \alpha^j) E_t \left( I_{t+1} c_{t+1}^j \right) + \frac{\text{cov} \left( r_{t+1}, r_{t+1} \right)}{\text{cov} \left( r_{t+1}, r_{t+1} \right) - \left( 1 + \alpha^M \right) I_{t+1} c_{t+1}^j} \left( I_{t+1} c_{t+1}^j - I_{t+1} c_{t+1}^j \right) \]

\[ + \frac{\text{cov} \left( c_{t+1}^j, r_{t+1} \right)}{\text{cov} \left( r_{t+1}, r_{t+1} \right) - \left( 1 + \alpha^M \right) I_{t+1} c_{t+1}^j} \left( I_{t+1} c_{t+1}^j - I_{t+1} c_{t+1}^j \right) \]

The model and proposition with empirical relevance would come if the liquidity level item and liquidity covariance item in equation (7) could be analyzed thus:

1. Liquidity level premium

From iteration expectation law, \( (1 + \alpha^j) E_t \left( I_{t+1} c_{t+1}^j \right) = \lambda_{t+1}^j (1 + \alpha^j) E_t \left( c_{t+1}^j | I_{t+1} = 1 \right) \) is established. So, the extent of influence for estimated return from estimated liquidity level \( E_t \left( c_{t+1}^j | I_{t+1} = 1 \right) \) depends on the stress degree of investor's liquidity demand, namely \( \lambda_{t+1}^j \), and the shock cost elasticity of the security itself \( \alpha^j \) in the next period.

2. Market liquidity systematic risk

\[ \text{cov} t_{1,1} \left( c_{t+1}^j, r_{t+1} \right), \text{cov} t_{1,1} \left( r_{t+1}, r_{t+1} \right), \text{cov} t_{1,1} \left( c_{t+1}^j, r_{t+1} \right) \]

It is in line with other's works that market liquidity systematic risk means the un-diversifiable risk. We need to separate it from the total risk by the iteration expectation law. The first item of the decomposition is presented thus:

\[ \text{cov} t_{1,1} \left( I_{t+1} c_{t+1}^j, I_{t+1} c_{t+1}^j \right) = \lambda_{t+1}^j \text{cov} t_{1,1} \left( c_{t+1}^j, c_{t+1}^j \right) + \left( \lambda_{t+1}^j - \lambda_{t+1}^j \right) E_t,_{1,1} c_{t+1}^j E_t,_{1,1} c_{t+1}^j \]

(9)

\[ \text{cov} t_{1,1} \left( r_{t+1}^j, I_{t+1} c_{t+1}^j \right) = \lambda_{t+1}^j \text{cov} t_{1,1} \left( r_{t+1}^j, c_{t+1}^j \right) + \lambda_{t+1}^j E_t,_{1,1} c_{t+1}^j \left( E_t,_{1,1} r_{t+1}^j - E_t,_{1,1} r_{t+1}^j \right) \]

(10)

\[ \text{cov} t_{1,1} \left( I_{t+1} c_{t+1}^j, r_{t+1}^j \right) = \lambda_{t+1}^j \text{cov} t_{1,1} \left( c_{t+1}^j, r_{t+1}^j \right) + \lambda_{t+1}^j E_t,_{1,1} c_{t+1}^j \left( E_t,_{1,1} r_{t+1}^j - E_t,_{1,1} r_{t+1}^j \right) \]

(11)
Here, the meaning of the expectation and covariance of $I = 1$ with subscript is the conditional expectation and conditional covariance when $I_{t+1} = 1$. For simplicity, we omit the subscript of $I$ below here.

$\text{cov}_{I_{t+1}}(c^i_{t+1}, c^M_{t+1})$, $\text{cov}_{I_{t+1}}(r^i_{t+1}, c^M_{t+1})$, and $\text{cov}_{I_{t+1}}(c^i_{t+1}, r^M_{t+1})$

are the liquidity systematic risk measurement. They are the covariance of return and liquidity of stock and that of market portfolio. Substituting (9), (10) and (11) into (7) can help us to analyze the relation between liquidity systematic risk and expected return. Based on this discussion, the study establishes the lemma 1.

Lemma 1: Both the systematic total risk and unit price are larger than zero, with the unit price

$$\pi_t = \frac{R}{\text{cov}(r^M_{t+1} - I_{t+1}c^M_{t+1}, r^M_{t+1} - (1 + \alpha^M)I_{t+1}c^M_{t+1})} > 0$$

lies in Appendix

(Proof)

Lemma 1 tells us the relation, three covariance and expected return is determined by the plus or minus sign in equation (7).

(1) $\text{cov}_{I_{t+1}}(c^i_{t+1}, c^M_{t+1})$ portrays the general character of the market. A stock with stronger general character would do less in diversifying risk of market total liquidity decrease and general recession. So it is the increase function for the relation of the market general character and the required return.

(2) $\text{cov}_{I_{t+1}}(r^i_{t+1}, c^M_{t+1})$ is the covariance of security's return and market portfolio liquidity. From equation (7) and (10), and Lemma 1, a security with larger covariance lead to a smaller required return. The reason is that a security with larger covariance can supply a better return in recession and protect investor from investment loss. So, a security with larger covariance means a lower required return and higher price.

(3) $\text{cov}_{I_{t+1}}(r^i_{t+1}, c^M_{t+1})$ is the covariance of liquidity of security and return of market portfolio. From equations (7), (11) and Lemma 1, a bigger $\text{cov}_{I_{t+1}}(r^i_{t+1}, c^M_{t+1})$ means a lower required return. The reason is that this kind of security would benefit a smaller transaction cost when total market suffers a drop. So investor desires to hold this kind of security to diversify market drop risk and ask for a lower return.

The basic idea is as the same of Acharya and Pedersen (2005) because the three covariance of this study have the subscript $I = 1$ but they do not. The meaning of $I = 1$ is that investor care more on the covariance of security and market when liquidity demand is tightening. We argue that whether the liquidity demand of investor is independent of the covariance of security and market, investor would care more on the market liquidity when the demand become more tense. A more tense liquidity demand of investor has a greater probability to liquidate his asset into cash, so he shows a stronger preference on liquidity and asks for a higher level to diversify his risk, and has a higher risk premium. The probability $\hat{\lambda}_{t+1}$ portrays the tension level, so the required unit premium of the liquidity systematic risk has the increasing relation with the probability $\hat{\lambda}_{t+1}$. Substitute equations (9), (10) and (11) into equation (7), we can get the unit premium equals to $\hat{\lambda}_{t+1} \Pi_t$. So, the time-varying of risk premium is led by the time-varying of $\hat{\lambda}_{t+1}$. Based on this discussion, the study establishes the lemma 2:

Lemma 2: The shock cost elasticity $\alpha^i$ would influence the difference of expected return between stocks. The liquidity premium would be influenced by $\hat{\lambda}_{t+1}$, and the unit price of systematic risk equals to $\hat{\lambda}_{t+1} \Pi_t$, so the risk premium would change with the adjustment of $\hat{\lambda}_{t+1}$.

RESULTS

The comparison of models

This model is an expansion of the model of Acharya and Pedersen (2005), they have inner link: if there is no friction and no liquidity premium in the market, equation (7) would degenerate to the classical CAPM. If liquidity problem exists in the market, but there is no endogenous liquidity risk and no uncertainty in liquidity demand state, correspondingly, the shock cost elasticity coefficient $\alpha^i = 0$. At this time, indicative function $I_{t+1}$ does not exist, equation (7) would degenerate into the model of Acharya and Pedersen’s (2005) liquidity risk pricing model. So, from empirical relevance, classical CAPM and the model of Acharya and Pedersen (2005) would be covered in the model of this study. The relevance is as follows:

1. Both our model and the model of Acharya and Pedersen can explain the premium effect phenomenon of $\text{cov}(c^i, c^d)$, $\text{cov}(r^i, c^M)$ and $\text{cov}(c^i, r^M)$ discovered from empirical study by Acharya and Pedersen (2005), Pastor and Stambaugh (2003). Furthermore, our model can explain what their model cannot explain.

They add the coefficient $\kappa^4$ before liquidity level $E(c^p_r)$ in their empirical model, but does not calibrate $\kappa$ into 1 from the theoretical model, instead, they take use of average monthly turnover rate as the calibration value of $\kappa$, which is in accord with the idea that probability $\hat{\lambda}$ would influence liquidity premium. For example, they separate all stocks into 25 portfolios on liquidity level to do regression analysis, $\kappa$ is calibrated to be 0.034. In reality, this method equals to a prior estimate of the probability of monthly selling of security, which equals to $\hat{\lambda}$ in our model. Strictly, the probability of security’s selling is bigger than $\hat{\lambda}$, because selling out securities do not mean to incur transaction cost loss.

Besides, in different grouping test, the estimation of $\kappa$ is significantly larger than the value which is calibrated on mean turnover. Based on the level of liquidity, the
calibrated value of different portfolios has the value of $\kappa$ as 0.034 in regression analysis, while that of estimated value is 0.042 (t-statistic is 2.21). Based on the uncertainty level, the calibrated value of different portfolios has the value of $\kappa$ as 0.035 in regression analysis, while that of estimated value is 0.062 (t-statistic is 2.433). Based on price-weighted investment portfolio and market portfolio, the calibrated value of $\kappa$ is 0.046 in robust regression test, while that of estimated value is 0.062 (t-statistic is 3.878). Based on capital-weighted investment portfolio and market portfolio, the calibrated value of $\kappa$ is 0.034 in robust regression test, while that of estimated value is 0.081 (t-statistic is 2.755). Based on the amount of investment portfolio and market portfolio, the calibrated value is 0.047 and the estimated value is 0.056, with the t-statistic equals to 2.139. Based on the amount of B/M, firstly grouping for individual stock, then set the sub-group in the group to get the calibrated value in regression analysis can supply the value of $\kappa$ as 0.045. At that time, estimated value is 0.167 and t-statistic is 3.452. It is the phenomenon which cannot get the answer from Acharya and Pedersen's model, but we can use our model to supply the solution. The calibrated value of $\kappa$ equals approximately to the estimated value of $\lambda$. In reality, the estimated value of $\kappa$ is the estimation of $(1 + \alpha)$, so all the estimated values are bigger than calibrated value, which is the strong proof that both $\alpha$ and $\lambda$ influence liquidity premium together.

2. Our model can explain the time-varying phenomenon of liquidity premium in Literature Review, but both the classical CAPM model and the model of Acharya and Pedersen cannot explain this phenomenon (Gibson and Mougeot 2004; Fujimoto and Watanabe 2005). Gibson and Mougeot (2004) are the first people who prove that our model indicates, and we supply a formal theoretical model. They discover empirically that the stock with high volatility and high turnover ratio has a high liquidity $\beta$ and significant liquidity risk premium, while stock with low volatility and low turnover ratio has a low liquidity $\beta$ and insignificant liquidity risk premium. We can explain this phenomenon, because volatility and turnover ratio are proxy ratio which is used as investor's liquidity preference, which equals to $\text{cov}_{t-1}(r_{t,1}, c_{t-1})$ in our model. So they discover that in different liquidity preference or different tension level, there is big dissimilarity in investor requiring risk premium. Furthermore, our model also supplies theoretical basis for their empirical result.

**Why is positive testing of liquidity risk premium’s time variant better?**

**Testing equation**

In order to test liquidity risk premium’s time variant, we can construct a regression testing equation based on equation (7):

$$r_{t+1} - r_t = a_t + \left( k_0 + k_1 dM1_t + k_2 dLEAD_t + k_3 dCHIBOR_t + k_4 dVOL_t \right) \text{illiq}_t$$

$$+ \left( b_0 + b_1 dM1_t + b_2 dLEAD_t + b_3 dCHIBOR_t + b_4 dVOL_t \right) \beta_{134,j}$$

$$+ b_{3t} \beta_{1,j} + b_{6t} \text{SIZE}_t + b_{7t} M / B_t + \epsilon_{t+1}.$$

(12)

In this equation, $\text{illiq}_t$ stands for the value of illiquidity in month $t$ and it is generated from price’s amplitude and the rate of trade volume. In detail, it is stock $j$’s price amplitude ((Day’s Maximum Price – Day’s Minimum Price) / Day’s Opening Price) in month $t$ divided by month’s average trade volume per day. Market Portfolio’s illiquidity index is the arithmetic mean of sample stock’s daily illiquidity index. $dM1_t$ stands for first order difference of money supply M1’s growth rate, $dLEAD_t$ stands for first order difference of economic and financial environment’s leading index, $dCHIBOR_t$ stands for first order difference of inter-bank borrowing weighted average monthly rate index, and $dVOL_t$ stands for stock market’s monthly volatility. We use these 4 variables as tool variables to capture liquidity premium. Since the growth rate of money supply M1, leading index LEAD, and inter-bank borrowing weighted average rate index CHIBOR’s monthly index serial all obey single root process, we implement differential processing on these variables. In theory, the correlation between these four variables and liquidity
premium (that is, required return) are showed in Table 1.

\[
\beta_{t+4}^{ij} = \left( \beta_{t}^{ij} - \beta_{t-1}^{ij} - \beta_{t}^{ij} \right)
\]
is the net beta of stock j's liquidity, in which

\[
\beta_{t}^{ij} = \frac{\text{cov}_t \left( c_{t+1}^j, c_{t+1}^M \right)}{\text{var}_t \left( r_{t+1}^M \right)};
\]

\[
\beta_{t-1}^{ij} = \frac{\text{cov}_{t-1} \left( c_{t}^j, c_{t}^M \right)}{\text{var}_{t-1} \left( r_{t}^M \right)};
\]

\[
\beta_{t}^{ij} = \frac{\text{cov}_t \left( c_{t}^j, c_{t}^M \right)}{\text{var}_t \left( r_{t}^M \right)}.
\]

\[
\beta_{t}^{ij} = \text{stock j's market beta, in which}
\]

\[
\beta_{t}^{ij} = \frac{\text{cov}_t \left( r_{t+1}^M, r_{t+1}^j \right)}{\text{var}_t \left( r_{t+1}^j \right)}.
\]

We also added two controlled variables \( \text{SIZE}_t \) and \( \text{M/B}_t \), they are respectively stock j's total trade value scale and price to book ratio on the market close time at the end of month t.

In the equation, the coefficients to be estimated are: \( a_0 \), \( k_0 \) to \( k_4 \), \( b_0 \) to \( b_4 \), \( b_{3n} \), and \( b_{7n} \). Coefficients \( k_0 \) to \( k_4 \) and \( b_0 \) to \( b_4 \) are used to capture the level of liquidity and risk premium's time variant. If \( b_1 \) to \( b_4 \) are not significant statistically, then our time variant proposition is not supported by statistics. If some coefficients among \( b_1 \) to \( b_4 \) are significantly not 0, we will say risk premium changes due to some tool variants' change. That means time variant proposition is supported by statistics. In same argument, whether \( k_1 \) to \( k_4 \) are significant or not means whether time variant proposition is supported by statistics.

The regression estimate method adopted is SUR method, one of the pool estimate methods. We adjusted disturbance term's period heteroscedasticity and autocorrelation in the estimation. When we are making the estimation based on stock's cross section and time serial statistics, \( k_0 \) to \( k_4 \) and \( b_0 \) to \( b_4 \) are all fixed coefficients to be estimated. However, other coefficients to be estimated (including \( a_0 \), \( b_{3n} \), \( b_{7n} \) and \( b_{7n} \)) are time variant. That means from January 1997 to December 2005, they have a estimation value every month \( \hat{a}_t \), \( \hat{b}_{3t} \), \( \hat{b}_{7t} \) and \( \hat{b}_{7t} \). In order to test these coefficients' significance, we calculated their average \( \bar{a} \), \( \bar{b}_3 \), \( \bar{b}_7 \), and \( \bar{b}_7 \) on the time serial and the corresponding t statistics as we have obtained estimation values' time serial. Finally, we use t statistics to test whether these coefficients are significantly not 0.

### Statistics

1) Sample statistic: Select Shanghai and Shenzhen stock markets' daily trading value statistics and financial statistics from January 3rd 1995 to December 30th 2005 in "www.wind.com.cn" ("Wind Info"). Considering illegal transactions such as manipulation, related transactions, and banker bucketing have long existed in China's stock markets, statistics of the manipulated and banker held stocks does not show real liquidity. So this article excluded ST stock, PT stock and long term suspended stocks in the analysis. Moreover, since China's stock market entered bear market for a long time since 2001, some banker held stocks “pledged” due to banker's bankruptcy or public exposure. As a result, we excluded the stocks that were exposed by media or showed three limit downs without company's general operation crisis or general market index's pledges. We also excluded stock's first month trade statistics after IPO. Finally, if some month's trading days is less than 15 days, we will exclude this month's statistics. In the analysis period, 136 ~ 912 stocks met the conditions above. These stocks represent the market portfolio of various periods. In order to make sure that the testing results have better reliability, we would like to select stocks which have longer public time in the time serial analysis. So the 136 stocks which went public before 1996 are selected finally. We will use these stocks' variable indexes such as monthly return, monthly liquidity and risk to in the econometric analysis so as to test our proposition.

2) Money supply M1 and inter-bank borrowing weighted-average rate (CHIBOR)'s monthly index statistics all come from "www.wind.com.cn" ("Wind Info"). Leading index (LEAD)'s monthly index statistics come from National Bureau of Statistics of China's website. Monthly volatility index (VOL) are calculated from our market portfolio return, that is, the every month's sample variation of market portfolio’s daily return.

### Model test results

Table 2 provides the estimation results of regression test equation (12). This estimation used 10 situations (10 models) to conduct the regression analysis. All estimations
of the goodness of fit $R^2$ of the 10 models exceed 75%. First analyze the result of model 1: estimation values of money supply ($dM_1$), leading index ($dLEAD$)'s coefficients $k_1$ and $k_2$ are both significant. Moreover, $k_1$ and $k_2$ are both negative. These results meet our forecast, i.e. when money supply is higher and future economy is better, the required return of unit illiquidity is lower. Nevertheless, China’s inter-bank borrowing rate ($dCHIBOR$) and market volatility ($VOL$)'s coefficient estimation is not significant in model 1. We also conduct liquidity risk premium time variant regression test on the 4 tool variants individually. That is, models 1 to 5. The results show that the coefficients’ estimations of money supply and leading index are significant (models 2 and 3). The signs of the estimation values are also the same as our theoretical forecast. Volatility coefficient’s estimation also becomes significantly positive (model 5). This obeys our theoretical forecast. However, the inter-bank borrowing rate is still not significant. We can see that in the 4 tool variants, two macroeconomic variables – money supply and leading index – most significantly affect the compensation return of per unit’s illiquidity; volatility ranks behind while inter-bank borrowing rate does not have any influence. We notice that, models 1 and 5 both consider regression estimation of illiquidity variant (that is, $illiq_j^t$) and liquidity risk variant (that is, $illiq_j^t plus$).

From the estimation results of the 5 models we can see that, due to the problem of collinearity, when illiquidity variant exists, the estimation value of liquidity risk variant either is not significant or has a sign that conflicts with our theoretical forecast. Only $b_1$’s estimation value obeys our theoretical forecast. So we are not able to effectively test the proposition that liquidity risk compensation is position. That means the existence of liquidity variant will affect liquidity risk coefficient’s estimation. Nevertheless, the existence of liquidity risk variant does not affect liquidity coefficient’s estimation. In order to avoid collinearity problem, we delete liquidity risk variant and merely conduct estimation on liquidity risk variant in models 6 and 10 so as to test liquidity risk premium’s time variant. From models 6 to 10, $b_1$’s estimation value grows bigger and becomes a significant positive value. We can see that when collinearity does not exist, the proposition that liquidity risk premium is positive can be effectively proved. Then, let’s turn to tool variants’ coefficient estimation. Models 6 and 7 show that whether other tool variants exist or not, money supply coefficient $b_1$ is significantly negative. This perfectly obeys theoretical forecast that unit liquidity risk premium decreases as money supply increases. However, other tool variants’ estimations fail to meet forecast result. Leading index coefficient’s estimation result is not negative as we expect (models 6 and 8). Market volatility is not always positive (models 6 and 10). China’s inter-bank borrowing rate coefficient’s estimation result totally opposes our expectation (models 6 and 9). Inter-bank borrowing rate’s result may result from the inaction of China’s inter-bank borrowing market. In an inactive market, the scarcity of capital does not strongly correlate with market rate. Thus, inter-bank market and capital market fail to act confirmatively.

In addition, the estimation of stock market beta coefficient is not significant. This means pure price risk does not ask for risk compensation from Chinese stock market. In short, liquidity and unit liquidity risk premium’s time variant proposition passes real economic statistics’ significance test. Money supply, leading index and market volatility are able to explain illiquidity compensation’s time variant, while money supply can better capture liquidity risk premium’s time variant.

Comparatively, money supply M1’s variance affects China’s stock market’s liquidity premium (including illiquidity compensation and liquidity risk premium) more confirmatively and significantly. Moreover, average of $k_1$’s two coefficients’ estimations is -0.12. Its absolute value is smaller than the absolute value of $b_1$’s 7 coefficients’ estimations’ average, which is -0.21. So, on average, money supply is more influential on unit liquidity risk premium than on unit liquidity premium. Multiply $dM_1$’s observed data, interval [-0.88, 1], by -0.21, we will get [-1.55, 1.75]. However, average of all sample stock’s illiquidity index ($illiq_j^t$) is 0.88 and average of liquidity beta ($\beta_j^t$) is 0.24. So we can roughly estimate that, due to stocks’ illiquidity and liquidity risk compensation return, monthly money supply variance contributes to stock’s monthly return variance at a level of about [-1.14, 1.30] on average in China. The calculation method is adding average illiquidity risk return contribution interval [-0.77, 0.88] to average liquidity risk return contribution interval [-0.37, 0.42], in which -0.77=-0.88*0.88, 0.88=1*0.88, -0.37=-1.55*0.24, 0.42=1.75*0.24. If we want the annual return rate, we can multiply it by 12. Based on this analysis, we can conclude that money supply has a great influence on stock market comparatively.

Conclusion

Liquidity and its risk are usually ignored in classical asset pricing equilibrium model. But in reality, liquidity risk is one of the prime risks for investors, especially institutional investors. In asset pricing model, rational investor would ask for appropriate risk compensation. In this paper, a liquidity risk-adjusted asset pricing model is given and the systematic part of liquidity risk would affect asset price and expected return. At the same time, we prove that investor would ask for premium for expected liquidity. The significance of liquidity risk premium would be proved by some empirical paper for the past few years, and our paper can help us understand the pricing
Table 2. Estimation results of regression test of equation (12).

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
<th>Model 7</th>
<th>Model 8</th>
<th>Model 9</th>
<th>Model 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.525 (0.65)</td>
<td>0.637 (0.80)</td>
<td>0.628 (0.79)</td>
<td>0.646 (0.808)</td>
<td>0.588 (0.73)</td>
<td>0.933 (1.17)</td>
<td>1.024 (1.28)</td>
<td>1.055 (1.31)</td>
<td>0.991 (1.22)</td>
<td>1.026 (1.28)</td>
</tr>
<tr>
<td>$k_0$</td>
<td>0.002 (0.02)</td>
<td>0.376* (9.32)</td>
<td>0.432* (10.40)</td>
<td>0.410* (10.13)</td>
<td>0.198* (3.97)</td>
<td>0.933 (1.17)</td>
<td>1.024 (1.28)</td>
<td>1.055 (1.31)</td>
<td>0.991 (1.22)</td>
<td>1.026 (1.28)</td>
</tr>
<tr>
<td>$k_1$</td>
<td>-0.120* (-8.97)</td>
<td>-0.119* (-9.15)</td>
<td>-0.358* (-3.45)</td>
<td>-0.496* (-4.92)</td>
<td>0.002 (0.02)</td>
<td>0.376* (9.32)</td>
<td>0.432* (10.40)</td>
<td>0.410* (10.13)</td>
<td>0.198* (3.97)</td>
<td>0.933 (1.17)</td>
</tr>
<tr>
<td>$k_2$</td>
<td>-0.010 (-0.08)</td>
<td>-0.029 (-0.25)</td>
<td>-0.010 (-0.08)</td>
<td>-0.029 (-0.25)</td>
<td>0.933 (1.17)</td>
<td>1.024 (1.28)</td>
<td>1.055 (1.31)</td>
<td>0.991 (1.22)</td>
<td>1.026 (1.28)</td>
<td>0.072* (6.02)</td>
</tr>
<tr>
<td>$k_3$</td>
<td>-0.010 (0.00)</td>
<td>-0.029 (-0.25)</td>
<td>-0.010 (0.00)</td>
<td>-0.029 (-0.25)</td>
<td>0.933 (1.17)</td>
<td>1.024 (1.28)</td>
<td>1.055 (1.31)</td>
<td>0.991 (1.22)</td>
<td>1.026 (1.28)</td>
<td>0.072* (6.02)</td>
</tr>
<tr>
<td>$k_4$</td>
<td>-0.255 (-0.81)</td>
<td>-0.671* (-2.23)</td>
<td>-1.21* (-3.99)</td>
<td>-1.219** (-4.04)</td>
<td>-1.094* (-3.64)</td>
<td>0.441* (1.70)</td>
<td>0.948* (7.80)</td>
<td>0.727* (6.36)</td>
<td>0.178* (1.71)</td>
<td>1.072* (6.84)</td>
</tr>
<tr>
<td>$k_5$</td>
<td>-0.080 (-1.55)</td>
<td>-0.092* (-1.87)</td>
<td>-0.358* (-7.68)</td>
<td>-0.308* (-6.89)</td>
<td>-0.261* (-5.74)</td>
<td>-0.276* (-6.16)</td>
<td>-0.095* (-3.78)</td>
<td>-0.095* (-3.78)</td>
<td>-0.095* (-3.78)</td>
<td>-0.095* (-3.78)</td>
</tr>
<tr>
<td>$k_6$</td>
<td>-0.496 (-0.98)</td>
<td>-1.611* (-3.40)</td>
<td>0.800 (1.65)</td>
<td>-0.542 (-1.36)</td>
<td>-0.390 (-0.98)</td>
<td>-0.375 (1.01)</td>
<td>-0.229 (0.679)</td>
<td>-0.229 (0.679)</td>
<td>-0.229 (0.679)</td>
<td>-0.229 (0.679)</td>
</tr>
<tr>
<td>$k_7$</td>
<td>-1.085* (-12.15)</td>
<td>-11.55* (-15.04)</td>
<td>-11.64* (-15.24)</td>
<td>-11.54* (-13.15)</td>
<td>-11.06* (-14.28)</td>
<td>-12.23* (-15.99)</td>
<td>-11.04* (-14.03)</td>
<td>-11.04* (-14.03)</td>
<td>-11.04* (-14.03)</td>
<td>-11.04* (-14.03)</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.080(1.34)</td>
<td>0.093* (1.89)</td>
<td>0.055(1.13)</td>
<td>0.088* (1.83)</td>
<td>0.004(0.08)</td>
<td>0.113* (2.32)</td>
<td>-0.029(-0.16)</td>
<td>-0.029(-0.16)</td>
<td>-0.029(-0.16)</td>
<td>-0.029(-0.16)</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.044(0.07)</td>
<td>-0.009(-0.01)</td>
<td>0.001(0.001)</td>
<td>0.001(0.001)</td>
<td>0.009(0.05)</td>
<td>-0.156(-0.26)</td>
<td>-0.136(-0.22)</td>
<td>-0.118(-0.193)</td>
<td>-0.122(-0.20)</td>
<td>-0.142(-0.23)</td>
</tr>
<tr>
<td>$b_3$</td>
<td>-0.046(-0.65)</td>
<td>-0.048(-0.67)</td>
<td>-0.047(-0.65)</td>
<td>-0.048(-0.66)</td>
<td>-0.048(-0.66)</td>
<td>-0.070(-0.96)</td>
<td>-0.079(-1.07)</td>
<td>-0.085-1.139</td>
<td>-0.082(-1.10)</td>
<td>-0.079(-1.05)</td>
</tr>
<tr>
<td>$b_4$</td>
<td>-0.055(-0.83)</td>
<td>-0.059(-0.90)</td>
<td>-0.060(-0.92)</td>
<td>-0.060(-0.92)</td>
<td>-0.064(-0.85)</td>
<td>-0.064(-0.85)</td>
<td>-0.069(-1.07)</td>
<td>-0.067(-1.04)</td>
<td>-0.063(-0.97)</td>
<td>-0.070(-1.08)</td>
</tr>
</tbody>
</table>

* shows that t test is significant at 5% level; the numbers in parentheses are t values.

mechanism of these phenomena. We agree with the basic ideas of liquidity risk premium from Acharya and Pedersen (2005). Besides that, the introduction of endogenous liquidity risk could let this paper keep with the reality that transaction cost is directly related with the volume of transaction position in the security market, especially in order-driven market. If the endogenous liquidity risk exists, the pricing shock cost elasticity of asset price would affect the expected return between assets. Another supplement is adding the liquidity demand of investor and randomizing the holding period. We reckon that the ability of diversification of market liquidity risk would be appreciated more when liquidity demand is tenser, and a higher risk premium would be incurred at this time. Our model could explain the time-varying phenomenon of liquidity risk premium better.

REFERENCES


APPENDIX

The proof of the proposition 1

As the utility function of investor is quadratic function, so random discount factor $m_{t+1}$ equals to:

$$m_{t+1} = \frac{U'\left(C_{t+1}\right)}{U'\left(C_t\right)} = \frac{\rho \left(\bar{C} - C_{t+1}\right)}{\rho \left(\bar{C} - C_t\right)}$$  \hspace{1cm} (A.1)

As the consumers are homogeneous, the proxy of investor’s portfolio is market portfolio, while the consumption level of investor depends on the net return of market portfolio. Restructure the constraint condition of consumption in equation (1) can lead to:

$$C_{t+1} = (r_{t+1}^M - I_{t+1}c_{t+1}^M)(C_t - C_t)$$  \hspace{1cm} (A.2)

We can get the lineal relation between random discount factor and net return of market portfolio:

$$m_{t+1} = a_t + b_t \left(r_{t+1}^M - I_{t+1}c_{t+1}^M\right)$$  \hspace{1cm} (A.3)

With $a_t = \rho \bar{C}/(\bar{C} - C_t)$, $b_t = -\rho(e_t - C_t)/(\bar{C} - C_t)$

From the result of equation (A.3), equation (3) can be simplified the relational expression of the asset return $r_{t+1}^f$:

$$1 = E_t\left(\left(a_t + b_t \left(r_{t+1}^M - I_{t+1}c_{t+1}^M\right)\right)\left(r_{t+1}^f - I_{t+1}c_{t+1}^f - I_{t+1}c_{t+1}^f\right)\right)$$ \hspace{1cm} (A.4)

$$\frac{\bar{R}_t}{\text{cov}_t\left(r_{t+1}^M - I_{t+1}c_{t+1}^M, r_{t+1}^M - I_{t+1}c_{t+1}^M\right)} = \frac{E_t\left(r_{t+1}^M - \left(1 + \alpha_t^M\right)I_{t+1}c_{t+1}^M - r_{t+1}^f\right)}{\text{cov}_t\left(r_{t+1}^M - I_{t+1}c_{t+1}^M, r_{t+1}^M - \left(1 + \alpha_t^M\right)I_{t+1}c_{t+1}^M\right)} > 0$$

The security $j$ here is any security in the market, so in a rational market, the pricing of any security would meet the equation above. Absolutely, risk-free asset $r_{t+1}^f$ and market portfolio $r_{t+1}^M$ also meet this equation. The coefficient $a_t$ and $b_t$ can be fixed by exploiting this fact, and system of simultaneous equations of $r_{t+1}^f$ and $r_{t+1}^M$ in the equation (A.4):

$$a_t = \frac{1}{r_{t+1}^f} - b_t E_t\left(r_{t+1}^M - I_{t+1}c_{t+1}^M\right)$$

$$b_t = -\frac{1}{r_{t+1}^f} \frac{E_t\left(r_{t+1}^M - I_{t+1}c_{t+1}^M\left(1 + \alpha_t^M\right) - r_{t+1}^f\right)}{\text{cov}_t\left(r_{t+1}^M - I_{t+1}c_{t+1}^M, r_{t+1}^M - \left(1 + \alpha_t^M\right)I_{t+1}c_{t+1}^M\right)}$$  \hspace{1cm} (A.5)

Substituting (A.5) into (A.4) and substituting $a_t$ and $b_t$ could get the result of the Proposition 1.

The proof of the Lemma 1

From the un-saturation condition of the consumer, we know $(\bar{C} - C_t) > 0$, so from the definition of $a_t$ and $b_t$ can lead to:

$$b_t < 0$$

And, $r_{t+1}^f$ is above 0, so from the equation of $b_t$ from (A.5) could get:

$$E_t\left(r_{t+1}^M - \left(1 + \alpha_t^M\right)I_{t+1}c_{t+1}^M - r_{t+1}^f\right) > 0$$

$$\text{cov}_t\left(r_{t+1}^M - I_{t+1}c_{t+1}^M, r_{t+1}^M - \left(1 + \alpha_t^M\right)I_{t+1}c_{t+1}^M\right) > 0$$